Paper Reference(s)

6686/01

Edexcel GCE

Statistics S4

Advanced/Advanced Subsidiary

Tuesday 16 June 2015 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Values from the statistical tables should be quoted in full. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P44828A

1. The Sales Manager of a large chain of convenience stores is studying the sale of lottery tickets in her stores. She randomly selects 8 of her stores. From these stores she collects data for the total sales of lottery tickets in the previous January and July. The data are shown below:

Store	A	B	C	D	E	F	G	H
January ticket sales (£)	1080	1639	710	1108	915	1066	1322	819
July ticket sales (£)	1113	1702	831	1048	861	1090	1303	852

(*a*) Use a paired *t*-test to determine whether or not there is evidence, at the 5% level of significance, that the mean sales of lottery tickets in this chain's stores are higher in July than in January. You should state your hypotheses and show your working clearly.

(8)

(*b*) State what assumption the Sales Manager needs to make about the sales of lottery tickets in her stores for the test in part (*a*) to be valid.

(1)

2. Fred is a new employee in a delicatessen. He is asked to cut cheese into 100 g blocks. A random sample of 8 of these blocks of cheese is selected. The weight, in grams, of each block of cheese is given below

94, 106, 115, 98, 111, 104, 113, 102

(*a*) Calculate a 90% confidence interval for the standard deviation of the weights of the blocks of cheese cut by Fred.

(6)

Given that the weights of the blocks of cheese are independent,

(*b*) state what further assumption is necessary for this confidence interval to be valid.

(1)

The delicatessen manager expects the standard deviation of the weights of the blocks of cheese cut by an employee to be less than 5 g. Any employee who does not achieve this target is given training.

(*c*) Use your answer from part (*a*) to comment on Fred's results.

(1)

A second employee, Olga, has just been given training. Olga is asked to cut cheese into 100 g blocks. A random sample of 20 of these blocks of cheese is selected. The weight of each block of cheese, x grams, is recorded and the results are summarised below.

$$\bar{x} = 102.6 \qquad s^2 = 19.4$$

Given that the assumption in part (*b*) is also valid in this case,

(*d*) test, at a 10% level of significance, whether or not the mean weight of the blocks of cheese cut by Olga after her training is 100 g. State your hypotheses clearly.

(6)

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3. As part of their research two sports science students, Ali and Bea, select a random sample of 10 adult male swimmers and a random sample of 13 adult male athletes from local sports clubs. They measure the arm span, x cm, of each person selected.

The data are summarised in the table below.

	n	s^2	$\bar{x}$
Swimmers	10	48	195
Athletes	13	161	186

The students know that the arm spans of adult male swimmers and of adult male athletes may each be assumed to be normally distributed.

They decide to share out the data analysis, with Ali investigating the means of the two distributions and Bea investigating the variances of the two distributions.

Ali assumes that the variances of the two distributions are equal. She calculates the pooled estimate of variance, s_p^2.

(*a*) Show that $s_p^2 = 112.6$ to 1 decimal place.

(2)

Ali claims that there is no difference in the mean arm spans of adult male swimmers and of adult male athletes.

(*b*) Stating your hypotheses clearly, test this claim at the 10% level of significance.

(5)

Bea believes that the variances of the arm spans of adult male swimmers and adult male athletes are not equal.

(*c*) Show that, at the 10% level of significance, the data support Bea's belief. State your hypotheses and show your working clearly.

(5)

Ali and Bea combine their work and present their results to their tutor, Clive.

(*d*) Explain why Clive is not happy with their research and state, with a reason, which of the tests in parts (*b*) and (*c*) is not valid.

(2)

4. A poultry farm produces eggs which are sold in boxes of 6. The farmer believes that the proportion, p, of eggs that are cracked when they are packed in the boxes is approximately 5%. She decides to test the hypotheses

$$H_0: p = 0.05 \quad \text{against} \quad H_1: p > 0.05.$$

To test these hypotheses she randomly selects a box of eggs and rejects H_0 if the box contains 2 or more eggs that are cracked. If the box contains 1 egg that is cracked, she randomly selects a second box of eggs and rejects H_0 if it contains at least 1 egg that is cracked. If the first or the second box contains no cracked eggs, H_0 is immediately accepted and no further boxes are sampled.

(*a*) Show that the power function of this test is

$$1 - (1 - p)^6 - 6p(1 - p)^{11}.$$

(3)

(*b*) Calculate the size of this test.

(2)

Given that $p = 0.1$,

(*c*) find the expected number of eggs inspected each time this test is carried out, giving your answer correct to 3 significant figures,

(3)

(*d*) calculate the probability of a Type II error.

(2)

Given that $p = 0.1$ is an unacceptably high value for the farmer,

(*e*) use your answer from part (*d*) to comment on the farmer's test.

(1)

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5. A researcher is investigating the accuracy of IQ tests. One company offers IQ tests that it claims will give any individual's IQ with a standard deviation of 5.

The researcher takes these tests 9 times with the following results:

123, 118, 127, 120, 134, 120, 118, 135, 121

(*a*) Find the sample mean, $\bar{x}$, and the sample variance, s^2, of these scores.

(2)

Given that any individual's IQ scores on these tests are independent and have a normal distribution,

(*b*) use the hypotheses

$$H_0: \sigma^2 = 25 \text{ against } H_1: \sigma^2 > 25$$

to test the company's claim at the 5% significance level.

(4)

Gurdip works for the company and has taken these IQ tests 12 times. Gurdip claims that the sample variance of these 12 scores is $s^2 = 8.17$.

(*c*) Use this value of s^2 to calculate a 95% confidence interval for the variance of Gurdip's IQ test scores.

[You may use $P(\chi^2_{11} > 3.816) = 0.975$ and $P(\chi^2_{11} > 21.920) = 0.025$]

(2)

(*d*) Assuming that $\sigma^2 = 25$, comment on Gurdip's claim.

(1)

6. A random sample $X_1, X_2, X_3, ..., X_{2n}$ is taken from a population with mean $\frac{\mu}{3}$ and variance $3\sigma^2$. A second random sample $Y_1, Y_2, Y_3, ..., Y_n$ is taken from a population with mean $\frac{\mu}{2}$ and variance $\frac{\sigma^2}{2}$, where the X and Y variables are all independent.

A, B and C are possible estimators of μ, where

$$A = \frac{X_1 + X_2 + X_3 + Y_1 + Y_2}{2}$$

$$B = \frac{3X_1}{2} + \frac{2Y_1}{3}$$

$$C = \frac{3X_1 + 4Y_1}{3}$$

(*a*) Show that two of A, B and C are unbiased estimators of μ and find the bias of the third estimator of μ.

(5)

(*b*) Showing your working clearly, find which of A, B and C is the best estimator of μ.

(4)

The estimator

$$D = \frac{1}{k}\left(\sum_{i=1}^{2n} X_i + \sum_{i=1}^{n} Y_i\right)$$

is an unbiased estimator of μ.

(*c*) Find k in terms of n.

(3)

(*d*) Show that D is also a consistent estimator of μ.

(4)

(*e*) Find the least value of n for which D is a better estimator of μ than any of A, B or C.

(2)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	Store: A, B, C, D, E, F, G, H; Difference *July-Jan*: 33, 63, 121, –60, –54, 24, –19, 33	B1
	$\bar{d} = \frac{141}{8} = (\pm)17.625$	M1
	$s_d^{\ 2} = \frac{8}{7}\left(\frac{28241}{8} - 17.625^2\right) = 3679.4\ldots$ or $s_d^{\ 2} = \frac{1}{7}\left(28241 - \frac{141^2}{8}\right) = 3679.4\ldots$	M1
	To test $\text{H}_0 : \mu_d = 0$ against $\text{H}_1 : \mu_d > 0$ (o.e.)	B1
	Test stat $t = \frac{17.625 - 0}{\sqrt{\frac{3679.4\ldots}{8}}} = 0.8218\ldots$	M1A1cso
	Critical value, $t_7 = 1.895$	B1
	Not in critical region therefore insufficient reason to reject H_0 No significant evidence that on average stores sell more lottery **tickets** in **July** than in **January**	A1ft (8)
(b)	Need assumption that the underlying distribution of the **difference** in sales in July and in January is **normally distributed**.	B1 (1)
		Total 9
	Notes	
(a)	1st B1 for differences all correct (o.e.) 1st M1 attempt to find $\bar{d} = \frac{\sum \text{"their } d\text{"}}{8}$ 2nd M1 attempting s_d or $s_d^{\ 2}$ $\frac{1}{7}\left(\sum \text{"their } d^2\text{"} - \frac{\left(\sum \text{"their } d\text{"}\right)^2}{8}\right)$ 2nd B1 both correct in terms of μ or μ_d (allow a defined symbol) condone $\mu_{July-Jan}$ 3rd M1 for attempting the correct test statistic $\frac{\bar{d}}{s_d / \sqrt{8}}$ 1st A1cso awrt 0.822 with no errors. 3rd B1 alternate method, p value of 0.219. Allow 2.365 for 2-tail test Final A1 need conclusion in context, need tickets July and January, ft their test stat and critical value **NB** difference of 2 means test gains no marks	
(b)	B1 need **differences** to be normally distributed, not just normal distribution	

Question Number	Scheme	Marks
2. (a)	$n=8 \quad \sum x = 843 \quad \sum x^2 = 89211$ $\therefore \quad \bar{x} = 105.375$ $s^2 = \frac{8}{7}\left(\frac{89211}{8} - 105.375^2\right) = 54.2678\ldots$ or $s^2 = \frac{1}{7}\left(89211 - \frac{843^2}{8}\right) = 54.2678\ldots$	M1A1
	Confidence interval is given by $\frac{7\times 54.267\ldots}{14.067} < \sigma^2 < \frac{7\times 54.267\ldots}{2.167}$	M1B1
	$\therefore \quad 27.004\ldots < \sigma^2 < 175.299\ldots$ $5.1966\ldots < \sigma < 13.240\ldots$	M1d A1 (6)
(b)	Need to assume underlying **Normal** distribution for **weights** of blocks of cheese.	B1 (1)
(c)	Lower limit of CI is $> 5\ g$ suggests that **Fred** needs **training**.	B1ft (1)
(d)	To test $\quad H_0 : \mu = 100\ , \quad H_1 : \mu \neq 100 \quad (\mu > 100)$ where μ is the mean weight of blocks of cheese	B1
	Test statistic $\quad t = \frac{102.6 - 100}{\sqrt{\frac{19.4}{20}}} = 2.6399\ldots$	M1A1
	Critical value(s): $\ t_{19} = (\pm)1.729\ (1.328)$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1	A1ft
	Significant evidence that the **mean weight** of the **blocks** of cheese is not 100 g (more than 100g)	B1cso (6)
		Total 14
	Notes	
(a)	1^{st} M1 attempting s or s^2 $\qquad$ 1^{st} A1 awrt 54.3 2^{nd} M1 for $\frac{7s^2}{\chi^2}$ B1 14.067 & 2.167 3^{rd} M1d Dept on previous M mark. Rearranging leading to interval for σ- must square root A1 awrt 5.20 and 13.2 (allow 5.2) **NB** a correct interval gains full marks	
(c)	B1ft on their CI must have **Fred/He/employee (do not allow empoloyees)** and **training**. They must have an interval in part(a)	
(d)	1^{st} B1 Both hypotheses with μ. Allow one-tail 1^{st} M1 $\frac{102.6-100}{\frac{s \text{ or } s^2}{\sqrt{20}}}$ $\qquad$ 1^{st} A1 awrt 2.64 2^{nd} B1 allow *p* value of 0.0161 in place of critical value. CV must follow from H_1 2^{nd} A1ft a correct statement – do not allow contradicting non context statement. 3^{rd} B1cso need correct conclusion in context containing the words in bold from a fully correct solution. For one tail need “more than 100g”	

Question Number	Scheme	Marks
3. (a)	$s_p^{\ 2} = \frac{12\times161+9\times48}{13+10-2} = \frac{2364}{21} = 112.571\ldots = 112.6\ (1\text{dp})$	M1A1cso (2)
(b)	To test $\text{H}_0 : \mu_s = \mu_a$ against $\text{H}_1 : \mu_s \neq \mu_a$ (o.e.)	B1
	Test stat, $t = \pm\frac{195-186}{\sqrt{112.57\ldots\left(\frac{1}{10}+\frac{1}{13}\right)}} = \pm 2.016\ldots$ (awrt2.02)	M1A1
	Critical values, $t_{21} = (\pm)1.721$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1 Evidence of difference in mean **arm span** of adult male **swimmers** and adult male **athletes** or No evidence to support **Ali's claim**.	A1 (5)
(c)	To test $\text{H}_0 : \sigma_s^{\ 2} = \sigma_a^{\ 2}$ against $\text{H}_1 : \sigma_s^{\ 2} \neq \sigma_a^{\ 2}$	B1
	Test stat, $F_{12,9} = \frac{161}{48} = 3.354\ldots \left(\frac{1}{\text{F}_{12,9}} = \frac{48}{161} = 0.2981\ldots\right)$	M1A1
	Critical value, $F_{12,9} = 3.07\ (0.3257\ldots)$	B1
	In critical region, therefore significant evidence to reject H_0 and accept H_1 Evidence of difference in **variance** of arm span of adult male swimmers and adult male athletes or the data supports **Bea's belief**	A1cso (5)
(d)	Should do test for variance first as **equal variances** is necessary assumption for *t* test for means	B1
	but is **not supported** in (c), so result in **(b) is invalid**.	B1d (2) **Total 14**
	Notes	
(a)	M1 for $\frac{12\times161+9\times48}{13+10-2}$ A1cso need to get awrt112.57 **or** $\frac{2364}{21}$ then write 112.6	
(b)	M1 $\frac{195-186}{\sqrt{112.6\left(\frac{1}{10}+\frac{1}{13}\right)}}$ 2nd B1 alternate method, *p* value of 0.0566 in place of critical value Final A1 requires correct conclusion in context	
(c)	1st B1 allow $\text{H}_0 : \sigma_s = \sigma_a$ against $\text{H}_1 : \sigma_s \neq \sigma_a$ M1 allow $\frac{161^2}{48^2}$ if they write the formula down Final A1 requires correct conclusion	
(d)	1st B1 equal variances is necessary assumption (may be implied by saying not equal) 2nd B1d but not supported in (c)/(variances not equal) therefore (b) result invalid	

Question Number	Scheme	Marks
4. (a)	Power function $= \text{P}(\text{H}_0 \text{ rejected}) = \text{P}(X_1 \geq 2) + \text{P}(X_1 = 1) \times \text{P}(X_2 \geq 1)$ $= 1-(1-p)^6 - 6p(1-p)^5 + 6p(1-p)^5 \times (1-(1-p)^6)$ $= 1-(1-p)^6 - 6p(1-p)^5 + 6p(1-p)^5 - 6p(1-p)^{11}$ $= 1-(1-p)^6 - 6p(1-p)^{11}$	M1A1 A1cso (3)
(b)	Size of test is value of power function when $p = 0.05$ Size of test $= 1 - 0.95^6 - 6 \times 0.05 \times 0.95^{11} = 0.094268\ldots$ (awrt 0.0943)	M1A1 (2)
(c)	E[number of eggs inspected] $= 12 \times \text{P}(X_1 = 1) + 6 \times \text{P}(X_1 \neq 1)$ $= 12 \times 6 \times 0.1 \times 0.9^5 + 6 \times (1-(6 \times 0.1 \times 0.9^5))$ $= 8.1257\ldots$(awrt 8.13)	M1 A1 A1 (3)
(d)	$\text{P}(\text{Type II error} \mid p = 0.1) = 1 -$ (value of power function when $p = 0.1$) $\text{P}(\text{Type II error} \mid p = 0.1) = 1-(1-0.9^6 - 6 \times 0.1 \times 0.9^{11}) = 0.7197\ldots$ (awrt 0.720)	M1 A1 (2)
(e)	Prob of Type II error, accepting $p = 0.05$ when it is actually 0.1, unacceptably high, is large, therefore not a good **test**.	B1 (1)
		Total 11
	Notes	
(a)	M1 for $\text{P}(X_1 \geq 2) + \text{P}(X_1 = 1) \times \text{P}(X_2 \geq 1)$ or $1 - \left(\text{P}(X_1 = 0) + \text{P}(X_1 = 1) \times \text{P}(X_2 = 0)\right)$ oe or a correct line of working A1 a correct line of working before the final answer A1 fully correct solution no errors.	
(b)	M1 attempt to subst 0.05 into (a)	
(c)	M1 for $12 \times \text{P}(X_1 = 1) + 6 \times \text{P}(X_1 \neq 1)$ A1 $12 \times 6 \times p \times 0.9(1-p)^5 + 6 \times (1-(6 \times p \times (1-p)^5)$	
(d)	M1 $1-(1-(1-p)^6 - 6 \times p \times (1-p)^{11})$	
(e)	B1 idea that the Probability of a Type II error is **too high** or the power is too low so the **test is not good/powerful or test needs changing**	

Question Number	Scheme	Marks
5 **(a)**	$\bar{x} = \frac{\sum x}{n} = \frac{1116}{9} = 124$	B1
	$s^2 = \frac{9}{8}\left(\frac{138728}{9} - 124^2\right) = 43$	
	Or	
	$s^2 = \frac{1}{8}\left(138728 - \frac{1116^2}{9}\right) = 43$	B1
		(2)
(b)	Test stat	
	$\chi^2 = \frac{8 \times 43}{25} = 13.76$	M1A1
	Critical value	
	$\chi^2 = 15.507$	B1
	Therefore not in critical region, insufficient evidence to reject H_0	
	There is evidence at the 5% level that the **company's** claim is supported	B1d
		(4)
(c)	CI given by	
	$\frac{11 \times 8.17}{21.920} < \sigma^2 < \frac{11 \times 8.17}{3.816}$	M1
	Therefore	
	$4.0999\ldots < \sigma^2 < 23.55\ldots$ awrt 4.10 and 23.6	A1
		(2)
(d)	$\sigma^2 = 25$ is not in CI which suggests **Gurdip's(his)** claim may not be true.	B1ft
		(1)
		Total 9
	Notes	
(a)	B1 124 B1 43	
(b)	M1 $\frac{8 \times \text{their } 43}{25}$ A1 awrt 13.8 B1 15.507 B1 dep on previous M1 being awarded. Allow the standard deviation of the **IQ** scores is 5 oe. Must have IQ	
(c)	M1 $\frac{11 \times 8.17}{3.816 \text{ or } 21.92}$ A1 both correct	
(d)	B1ft their interval from part(c). Gurdip's claim may not be true NB, no interval in (c) then B0	

Question Number	Scheme	Marks
6. (a)	$\text{E}[A]=\frac{1}{2}(\text{E}[X_1]+\text{E}[X_2]+\text{E}[X_3]+\text{E}[Y_1]+\text{E}[Y_2]=\frac{1}{2}\left(3\times\frac{\mu}{3}+2\times\frac{\mu}{2}\right)=\mu$	M1
	Therefore A is an unbiased estimator	A1
	$\text{E}[B]=\frac{3\text{E}[X_1]}{2}+\frac{2\text{E}[Y_1]}{3}=\frac{3}{2}\times\frac{\mu}{3}+\frac{2}{3}\times\frac{\mu}{2}=\frac{5\mu}{6}$	A1
	Therefore B is biased with bias $(-)\frac{\mu}{6}$	B1ft
	$\text{E}[C]=\frac{1}{3}\left(3\text{E}[X_1]+4\text{E}[Y_1]\right)=\frac{1}{3}\left(\frac{3\mu}{3}+\frac{4\mu}{2}\right)=\mu$	
	Therefore C is an unbiased estimator	A1
		(5)
(b)	Best estimator is unbiased estimator with least variance	
	$\text{Var}\left(A\right)=\frac{1}{4}(\text{Var }X_1+\text{Var }X_2+\text{Var }X_3+\text{Var }Y_1+\text{Var}Y_2)$	M1
	$=\frac{1}{4}\left(3\times3\sigma^2+2\times\frac{\sigma^2}{2}\right)=\frac{5\sigma^2}{2}$	A1
	$\text{Var}\left(C\right)=\frac{1}{9}(9\text{Var }X_1+16\text{Var }Y_1)=\frac{1}{9}\left(9\times3\sigma^2+16\times\frac{\sigma^2}{2}\right)=\frac{35\sigma^2}{9}$	A1
	Therefore A is a better estimator of μ (smaller variance)	B1dft
		(4)
(c)	$\text{E}[D]=\frac{1}{k}\left(2n\times\frac{\mu}{3}+n\times\frac{\mu}{2}\right)=\mu$	M1A1
	$k=\frac{2n}{3}+\frac{n}{2}=\frac{7n}{6}$	A1
		(3)
(d)	$\text{Var}\left(D\right)=\frac{1}{k^2}\left(2n\times3\sigma^2+n\times\frac{\sigma^2}{2}\right)=\frac{1}{k^2}\times\frac{13n\sigma^2}{2}$	M1
	$\text{Var}\left(D\right)=\frac{36}{49n^2}\times\frac{13n\sigma^2}{2}=\frac{234\sigma^2}{49n}$	M1d A1
	Therefore $\text{Var }D\to0$ as $n\to\infty$, therefore D is a consistent estimator	A1dd
		(4)
(e)	Want	
	$\frac{234\sigma^2}{49n}<\frac{5\sigma^2}{2}$	M1
	Therefore	
	$\frac{234}{49}\times\frac{2}{5}<n$	
	$n>1.910\ldots$	
	So minimum value is $n=2$	A1cso
		(2)
		Total 18

	Notes	
(a)	M1 for a correct method for E(A) or E(B) or E(C)	
	A1 for each correct expectation with a correct method	
	B1ft bias of B, condone missing – sign. Do not allow a bias of 0	
(b)	M1 Use of $\text{Var}(aX) = a^2\text{Var}(X)$ and subst $3\sigma^2$ for Var(X) and $\frac{\sigma^2}{2}$ for Var(Y)	
	A1 for each correct variance	
	B1dft their variances. Dep on m1 being awarded. If no variances given then B0	
(c)	M1 attempts E(D) and puts = to μ (may be implied)	
	A1 for E(D)	
(d)	M1 for $\frac{1}{k^2}\left(2n\times 3\sigma^2 + n\times\frac{\sigma^2}{2}\right)$ or $\frac{1}{k^2}\times\frac{13n\sigma^2}{2}$	
	M1d for subst in k	
	A1 Correct Var (D)	
	A1dd Need correct reason for being a consistent estimator dep on previous method marks being awarded	
(e)	M1 for forming an inequality with their Var(D) < their best estimator leading to n	